Paper Reference(s)

6678/01

Edexcel GCE

Mechanics M2

Advanced/Advanced Subsidiary

Tuesday 9 June 2015 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P44837A

1. A van of mass 900 kg is moving down a straight road that is inclined at an angle θ to the horizontal, where $\sin\theta = \frac{1}{30}$. The resistance to motion of the van has constant magnitude 570 N. The engine of the van is working at a constant rate of 12.5 kW.

At the instant when the van is moving down the road at 5 m s^{-1}, the acceleration of the van is a m s^{-2}.

Find the value of a.

(5)

2.

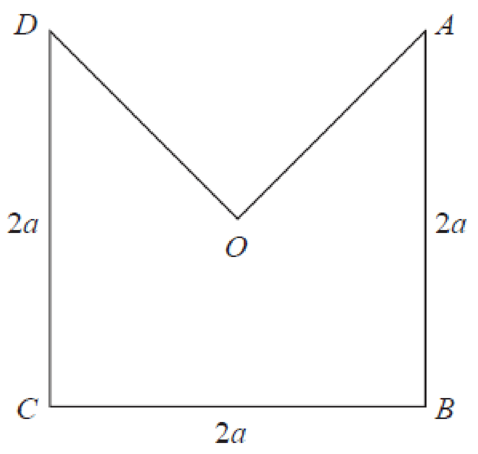

Figure 1

The uniform lamina *OABCD*, shown in Figure 1, is formed by removing the triangle *OAD* from the square *ABCD* with centre *O*. The square has sides of length 2*a*.

(*a*) Show that the centre of mass of *OABCD* is $\frac{2}{9}a$ from *O*.

(4)

The mass of the lamina is *M*. A particle of mass *kM* is attached to the lamina at *D* to form the system *S*. The system *S* is freely suspended from *A* and hangs in equilibrium with *AO* vertical.

(*b*) Find the value of *k*.

(4)

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3. A particle P of mass 0.75 kg is moving with velocity $4\mathbf{i}$ m s^{-1} when it receives an impulse $(6\mathbf{i} + 6\mathbf{j})$ N s. The angle between the velocity of P before the impulse and the velocity of P after the impulse is $\theta°$.

Find

(*a*) the value of θ,

(5)

(*b*) the kinetic energy gained by P as a result of the impulse.

(3)

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4. A ladder AB, of weight W and length $2l$, has one end A resting on rough horizontal ground. The other end B rests against a rough vertical wall. The coefficient of friction between the ladder and the wall is $\frac{1}{3}$. The coefficient of friction between the ladder and the ground is μ. Friction is limiting at both A and B. The ladder is at an angle θ to the ground, where $\tan\theta = \frac{5}{3}$. The ladder is modelled as a uniform rod which lies in a vertical plane perpendicular to the wall.

Find the value of μ.

(9)

5.

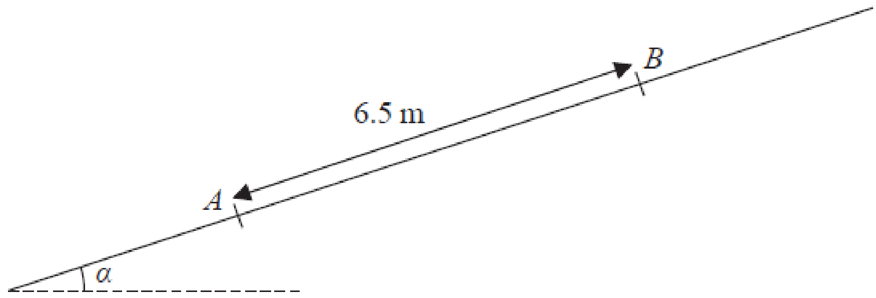

Figure 2

A particle P of mass 10 kg is projected from a point A up a line of greatest slope AB of a fixed rough plane. The plane is inclined at angle α to the horizontal, where $\tan \alpha = \frac{5}{12}$ and $AB = 6.5$ m, as shown in Figure 2. The coefficient of friction between P and the plane is μ. The work done against friction as P moves from A to B is 245 J.

(*a*) Find the value of μ.

(5)

The particle is projected from A with speed 11.5 m s^{-1}. By using the work-energy principle,

(*b*) find the speed of the particle as it passes through B.

(4)

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6. A particle P moves on the positive x-axis. The velocity of P at time t seconds is $(2t^2 - 9t + 4)$ m s^{-1}. When $t = 0$, P is 15 m from the origin O.

Find

(*a*) the values of t when P is instantaneously at rest,

(3)

(*b*) the acceleration of P when $t = 5$,

(3)

(*c*) the total distance travelled by P in the interval $0 \le t \le 5$.

(5)

7.

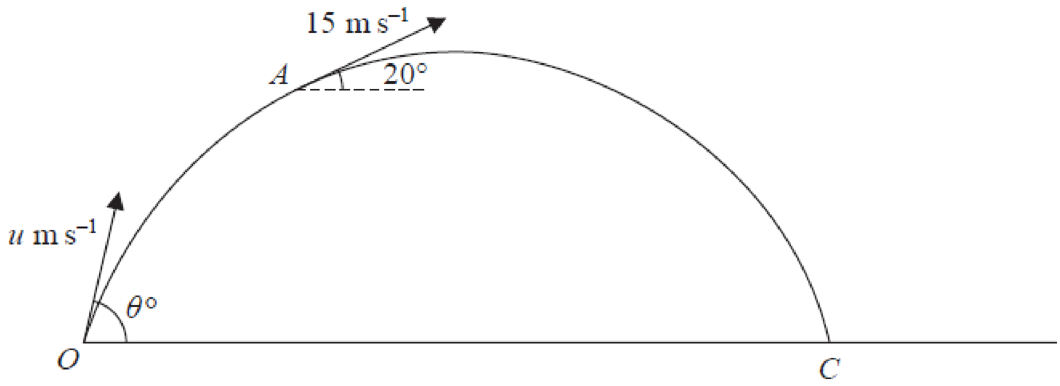

Figure 3

At time $t = 0$, a particle is projected from a fixed point O on horizontal ground with speed u m s^{-1} at an angle $\theta°$ to the horizontal. The particle moves freely under gravity and passes through the point A when $t = 4$ s. As it passes through A, the particle is moving upwards at 20° to the horizontal with speed 15 m s^{-1}, as shown in Figure 3.

(*a*) Find the value of u and the value of θ.

(7)

At the point B on its path the particle is moving downwards at 20° to the horizontal with speed 15 m s^{-1}.

(*b*) Find the time taken for the particle to move from A to B.

(2)

The particle reaches the ground at the point C.

(*c*) Find the distance OC.

(3)

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8. Three identical particles P, Q and R, each of mass m, lie in a straight line on a smooth horizontal plane with Q between P and R. Particles P and Q are projected directly towards each other with speeds $4u$ and $2u$ respectively, and at the same time particle R is projected along the line away from Q with speed $3u$. The coefficient of restitution between each pair of particles is e. After the collision between P and Q there is a collision between Q and R.

(*a*) Show that $e > \frac{2}{3}$.

(7)

It is given that $e = \frac{3}{4}$.

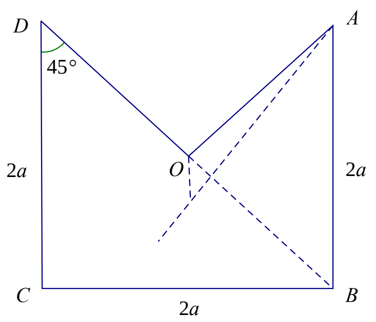
(*b*) Show that there will not be a further collision between P and Q.

(6)

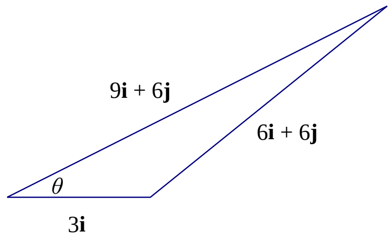
TOTAL FOR PAPER: 75 MARKS

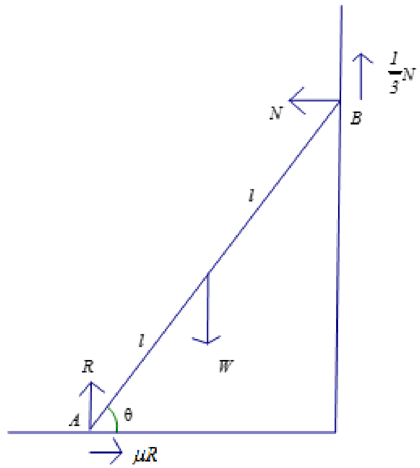
June 2015
6678 Mechanics 2
Mark Scheme

Question Number	Scheme	Marks	
1.	$12500 = 5F$	B1	Use of $P = Fv$
	$F + 900g\sin\theta - 570 = 900a$	M1	Use of $F = ma$ parallel to the slope. All 4 terms required. Condone sign errors and sin/cos confusion. 12500 in place of F is M0 – dimensionally incorrect
		A2	Correct unsimplified equation. -1 each error
	$a = 2.47 \quad (2.5)$	A1	
			Working with the positive direction up the slope is acceptable for the first 4 marks, but their final answer must be positive.
		[5]	

Question Number	Scheme	Marks	
2a	Ratio of masses 1 : 3 : 4	B1	Correct ratios for their division Also common: 3 equal triangles, 6 equal triangles, a rectangle and two equal triangles
	Centre of mass of triangle $\frac{2}{3}a$ from O	B1	Correct centres for the triangles in their division consistent with their axis.
	Moments about horizontal axis through O: $4\times 0-1\times\frac{2}{3}a=3d$	M1	Condone 4×0 not seen. Terms must be of correct form Condone use of moments about a parallel axis Signs must be consistent with their axis and distances. Watch out for people who add a triangle to the square.
	$\left(d=-\frac{2}{9}a\right)$ Distance = $\frac{2}{9}a$	A1	Reach ***Given answer*** with no errors seen. Their answer must be positive
		[4]	
2b	 		
	$\sqrt{2}akM=\frac{2}{9}a\cos 45M$	M1	Moments about A Lengths must be resolved as necessary (need use of trig).
		A2	-1 each error
	$k=\frac{1}{9}$	A1	
		[4]	

Alt 2b	Take *A* as origin and axes along *AD* and *AB*, use moments to find components of distance of c of m from *A*.	M1	Could choose a different origin
	$\bar{x} = \frac{a(1+2k)}{1+k}$	A1	
	$\bar{y} = \frac{11a}{9(1+k)}$	A1	
	$\bar{x} = \bar{y} \Rightarrow \frac{11}{9} = 1 + 2k \Rightarrow k = \frac{1}{9}$	A1	CWO. This mark is not available if they have assumed that the c of m of the system is at *O*.
Alt 2b	$\frac{2}{9}a\cos 45 = \frac{1}{9}.\sqrt{2}a = \frac{1}{9}OD$	M1A1	Using ratios
	$kM \times OD = M \times \frac{1}{9}OD$	A1	
	$\Rightarrow k = \frac{1}{9}$	A1	

Question Number	Scheme	Marks	
3a	$: \ 0.75\mathbf{v} = 6\mathbf{i} + 6\mathbf{j} + 0.75 \times 4\mathbf{i} \ \left(= 9\mathbf{i} + 6\mathbf{j}\right)$	M1	Impulse momentum equation Must be considering $\pm m\left(\mathbf{v} - \mathbf{u}\right)$
		A1	Correct unsimplified
	$\mathbf{v} = 12\mathbf{i} + 8\mathbf{j}$	A1	Award in (a) if seen or if (a) is completed correctly. Award in (b) if (a) is incomplete and this mark has not been awarded and correct **v** seen for the first time in (b)
			Could have a velocity triangle rather than momentum, in which case the vectors are $4\mathbf{i}, 8\mathbf{i} + 8\mathbf{j}, 12\mathbf{i} + 8\mathbf{j}$
	$\theta = \tan^{-1}\left(\frac{2}{3}\right)$ or $\theta = \cos^{-1}\left(\frac{1+13-8}{2\sqrt{13}}\right)$, or equivalent	M1	Correct trig to find the required angle
	$33.7°$ or 0.588 radians	A1	Accept $34°$ or better . Must be the final answer
		[5]	
3a alt	$\begin{pmatrix} 6 \\ 6 \end{pmatrix} = 0.75\begin{pmatrix} v\cos\theta \\ v\sin\theta \end{pmatrix} - 0.75\begin{pmatrix} 4 \\ 0 \end{pmatrix}$	M1A1	
	$\Rightarrow 0.75 \times v\cos\theta = 9, \quad 0.75 \times v\sin\theta = 6$	A1	
	$\Rightarrow \tan\theta = \frac{2}{3}$	M1	
	$33.7°$ or 0.588 radians	A1	
3b	Change in KE $= \frac{1}{2} \times \frac{3}{4}(144 + 64) - \frac{1}{2} \times \frac{3}{4}(16)$	M1	Finding a difference between KE terms. Must **use** $\frac{1}{2}mv^2$ in both terms (all of v, not just one component of it.)
		A1ft	follow their **v.** Allow $\pm$
	=72 (J)	A1	CAO
		[3]	
		(8)	

Question Number	Scheme	Marks	
4			NB: If μ and $\frac{1}{3}$ are used the wrong way round the candidate loses the first A1 and the final A1.
	Resolve horizontally or vertically:	M1	Allow without friction $=\mu R$
	$\mu R = N$ or $W = R + \frac{1}{3}N$	A1	With coefficient(s) of friction . Condone Wg
	Take moments about A or B.	M1	All terms required but condone sign errors and sin/cos confusion. Terms must be resolved.
	$M(A)$: $2lN\sin\theta + 2l\frac{N}{3}\cos\theta = Wl\cos\theta$ $M(B)$: $2l\cos\theta R = Wl\cos\theta + \mu R 2l\sin\theta$	A2	-1 each error. Could be in terms of Fs. -1 if see Wg in place of W. Any Friction force used should be acting in the right direction. Mark the equation, not what they have called it.
	$\frac{10}{3}N + \frac{2}{3}N = W$ or $2R = W + 2\mu R \times \frac{5}{3}$	M1	Use $\tan\theta = 5/3$ (substitute values for the trig ratios)
	$\Rightarrow 4N = W \Rightarrow 4N - R = \frac{1}{3}N$	DM1	Equation in N and R (Eliminate one unknown) Dependent on the moments equation
	$\frac{11}{3}\mu R = R$	DM1	Solve for μ Dependent on the moments equation
	$\mu = \frac{3}{11} (\simeq 0.273)$	A1	0.27 or better

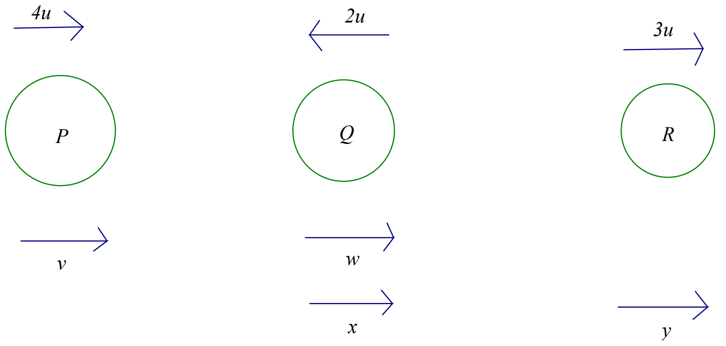
Alt 1	Resolve horizontally or vertically:	M1	Allow without friction $=\mu R$
	$\mu R = N$ or $W = R + \frac{1}{3}N$	A1	With coefficient(s) of friction
		M1	Take moments about *A* or *B*. All terms required but condone sign errors and sin/cos confusion. Terms must be resolved.
	$M(A): 2lN\sin\theta + 2l\frac{N}{3}\cos\theta = Wl\cos\theta$ $M(B):\ 2l\cos\theta R = Wl\cos\theta + \mu R 2l\sin\theta$	A2	-1 each error, Could be in terms of *F*s. -1 if *Wg* used. Mark the equation, not what they have called it. Any Friction force used should be acting in the right direction. For this method they need two moments equations – allows the marks for their best equation.
	$2lN\sin\theta + 2l\frac{N}{3}\cos\theta = 2l\cos\theta R - \mu R 2l\sin\theta$	DM1	Use two moments equations to eliminate *W* Dependent on the moments equation
	Use of $\tan\theta$: $2\mu\times\frac{5}{3}+\frac{2}{3}\mu = 2 - 2\mu\times\frac{5}{3}$	M1	Substitute for the trig ratios
	Solve for μ : $\left(\frac{20}{3}+\frac{2}{3}\right)\mu = 2$,	DM1	Dependent on the moments equation
	$\mu = \frac{3}{11}(\simeq 0.273)$	A1	0.27 or better

Alt 2	Resolving horizontally or vertically:	M1	Allow without friction $=\mu R$
	$\mu R = N$ or $W = R + \frac{1}{3}N$	A1	With coefficient(s) of friction (condone Wg)
	$l\cos\theta \times R = l\cos\theta \times \frac{1}{3}N + l\sin\theta \times N + l\sin\theta \times \mu R$	M1	Moments about the centre of the rod. All terms required. Terms must be resolved. Condone sign errors and sin/cos confusion. Allow without friction $= \frac{1}{3}N$. Any Friction force used should be acting in the right direction.
		A2	-1 each error. Could be in terms of Fs. -1 if Wg used.
	$l\cos\theta \times R = l\cos\theta \times \frac{1}{3}\mu R + l\sin\theta \times \mu R + l\sin\theta \times \mu R$	DM1	Obtain an equation in μ and θ $\left(\cos\theta = \cos\theta \times \frac{1}{3}\mu + \sin\theta \times \mu + \sin\theta \times \mu\right)$ Dependent on the moments equation
	$\cos\theta\left(1 - \frac{1}{3}\mu\right) = 2\mu\sin\theta \Rightarrow \tan\theta = \dfrac{1-\frac{1}{3}\mu}{2\mu} = \frac{5}{3}$	M1	Use of $\tan\theta$ (substitute values for the trig ratios)
	Solve for μ : $10\mu = 3 - \mu$,	DM1	Dependent on the moments equation
	$\mu = \frac{3}{11} (\simeq 0.273)$	A1	0.27 or better
		[9]	

Question Number	Scheme	Marks	
5a	Max friction $=\mu \times 10g\cos\alpha$	B1	Use of μR seen or implied $(\mu \times 90.46)$
	Work done against friction $=6.5\times 10g\mu\cos\alpha\,(=245)$	M1	For 6.5 x their *F*.
	Equation in μ : $6.5\times 10g\mu\times\frac{12}{13}=245$,	DM1	Dependent on preceding M1.
		A1	Correct substituted equation
	$\mu = 0.417$ or 0.42	A1	Do not accept $\frac{5}{12}$ (cannot have an exact value following the use of 9.8) Use of 9.81 is a rubric infringement, so A0 if seen.
		[5]	
5b	$\frac{1}{2}\times 10\times 11.5^2 - 245 - 10g\times 6.5\sin\alpha = \frac{1}{2}\times 10\times v^2$ or equivalent	M1	**Must be using work-energy equation**. All terms required. Condone sign errors and sin/cos confusion.
		A2	-1 each error $(661.25-245-245=171.25)$
	$v = 5.85$ (5.9) (m s^{-1})	A1	
		[4]	
		(9)	

Question Number	Scheme	Marks	
6a	At rest when $v=0$: $\left(2t^2-9t+4\right)=0$	M1	
	$=(2t-1)(t-4)$,	DM1	Solve for t. Dependent on the previous M1
	$t=\frac{1}{2},\ 4$	A1	Incorrect answers with no method shown score M0A0
		[3]	
6b	$a=\frac{\mathrm{d}v}{\mathrm{d}t}=4t-9$	M1	Differentiate *v* to obtain *a* (at least one power of *t* going down)
		A1	Correct derivative
	$t=5$, $a=11$ (m s^{-2})	A1	
		[3]	
6c	$s=\int v\mathrm{d}t=\frac{2}{3}t^3-\frac{9}{2}t^2+4t\ (+C)$	M1	Integrate *v* to obtain *s* (at least one power of *t* going up)
		A1	
	Use of $t=0$, $t=\frac{1}{2}$, $t=4$, $t=5$ (and $t=0, s=15$) as limits in integrals	DM1	Correct strategy for their limits - requires subtraction of the negative distance. Dependent on the previous M1 and at least one positive solution for *t* in (0,5) from (a)
	$\left[\frac{2}{3}t^3-\frac{9}{2}t^2+4t(+15)\right]_0^{\frac{1}{2}}-\left[\frac{2}{3}t^3-\frac{9}{2}t^2+4t(+15)\right]_{\frac{1}{2}}^4+\left[\frac{2}{3}t^3-\frac{9}{2}t^2+4t(+15)\right]_4^5$	A1	NB: $\int_0^5 v\mathrm{d}t$ scores M0A0A0
	(0, $\frac{23}{24}$, $-\frac{40}{3}$, $\frac{-55}{6}$) $=\frac{23}{24}+\frac{343}{24}+\frac{100}{24}=19.4$ (m) $\left(15,\ 15\frac{23}{24}\left(\frac{383}{24}\right),\ \frac{5}{3},\ 5.8\dot{3}\left(\frac{35}{6}\right)\right)$	A1 [5]	$19\frac{5}{12}$ $\left(\frac{233}{12}\right)$ or better
		(11)	

Question Number	Scheme	Marks	
7a	After 4 seconds from O, horizontal speed $= u\cos\theta$	B1	
	Vertical component of speed at $A = u + at$	M1	Complete method using *suvat* to find *v*.
	$= u\sin\theta - 4g$	A1	
	At *A*, components are $15\cos 20$ (horizontal) and $15\sin 20$ (vertical)	B1	
	$u\cos\theta = 15\cos 20$ $u\sin\theta = 15\sin 20 + 4g$	DM1	Form simultaneous equations in u and θ and attempt to solve for u or θ. Depends on the previous M1
	$\theta = 72.4 \quad (72)$	A1	Remember - A0 for the first overspecified answer
	$u = 46.5 \quad (47)$	A1	
		[7]	
Alt7a	After 4 seconds from O, horizontal speed $= u\cos\theta$	B1	
	At $t = 4$, $s = vt - \frac{1}{2}gt^2$	M1	Complete method to find the vertical height at *A*
	$= 98.9......$	A1	
	At A, components are $15\cos 20$ (horizontal) and $15\sin 20$ (vertical)	B1	
	$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 - 2gh$	DM1	Conservation of energy. The equation needs to include all three terms but condone sign error(s).
	$u = 46.5 \quad (47)$	A1	Remember - A0 for the first overspecified answer
	$\theta = 72.4 \quad (72)$	A1	Beware inappropriate use of *suvat*
7b	$-15\sin 20 = 15\sin 20 - gt$ or $0 = 15\sin 20t - \frac{1}{2}gt^2$	M1	Complete method using *suvat* or otherwise to find the time to travel from *A* to *B*
	$t = 1.05$ (s) or 1.0 (s)	A1	
		[2]	
7c	Total time $= 4 + (1.05) + 4$	B1ft	Follow their *t* or $\frac{2u\sin\theta}{g}$ for their u, θ
	Range = $46.5 \times \cos 72.4 \times (8 + 1.05)$ (or $15\cos 20 \times 9.05$)	M1	Correct method to find *OC* for their *t*,*u* and θ
	$= 128$ (m) or 127 (m) (130)	A1	
		[3]	
		(12)	

Question Number	Scheme	Marks	
8a			
	$4mu - 2mu = mv + mw$	M1	Equation for CLM. Requires all 4 terms. Condone sign errors. Condone m missing throughout.
		A1	
	$w - v = 6eu$	M1	Impact law. e must be used correctly. Condone sign errors
		A1	Signs should be consistent with equation for CLM.
	$v + w = 2u$ $w - v = 6eu$ $\quad 2w = 2u + 6eu$, $(w = u + 3eu)$	DM1	Solve for w. Dependent on the two previous M marks.
	For Q and R to collide require $w > 3u$,	M1	Use **inequality** to compare their w with $3u$.
	$u + 3eu > 3u$, $\quad e > \frac{2}{3}$	A1	Reach ***Given answer*** with no errors seen.
		[7]	

8a alt	Collision between Q and $R \Rightarrow w > 3u$	M1	
	Magnitude of impulse on $Q > 5mu$		
	Magnitude of impulse on $P > 5mu$	A1	
	$\Rightarrow v < -u$	M1	
	$e = \dfrac{w - v}{4u + 2u}$	M1	Impact law
	Speed of separation after collision $> u + 3u$	M1	
	$e > \dfrac{4u}{4u + 2u}$	A1	
	$e > \dfrac{2}{3}$	A1	Reach given inequality with no errors seen.
8b	$w = u + 3eu = \dfrac{13}{4}u, \quad v = -\dfrac{5}{4}u$	B1	Correct values seen or implied.
	$mw + 3mu = mx + my$ $\dfrac{3}{4}(w - 3u) = y - x$	M1	Equations for CLM and impact. Allow with w or their w. All terms required. Condone sign errors. e must be used correctly.
		A1	Correct equations in w or their w.
	$\dfrac{25}{4}u = x + y, \quad \dfrac{3u}{16} = y - x$		
	Solve for x (or kx)	DM1	Dependent on the previous M1 Need to get far enough to give a convincing concluding argument.
	$\dfrac{97}{16}u = 2x, \quad x = \dfrac{97u}{32}(= 3.03125u)$	A1	Correct expression for kx
	P and Q moving away from each other, so no collision.	A1	Reach **given conclusion** with no errors seen.
		[6]	
		(13)	